Valuation in Dynamic Bargaining Markets

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Abstract

We study the impact on asset prices of illiquidity associated with search and bargaining in an economy in which agents can trade only when they find each other. Marketmakers' prices are higher and bid-ask spreads are lower if investors can find each other more easily. Prices become Walrasian as investors' or marketmakers' search intensities get large. Endogenizing search intensities yields natural welfare implications. Information can fail to be revealed through trading when search is difficult.

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1 Introduction

In some markets, an investor who wants to sell an asset must search for a buyer, incurring opportunity or other costs until a buyer is found. When two counterparties meet, their bilateral relationship is inherently strategic. Prices are set through a bargaining process that reflects each investor's alternatives to immediate trade. The buyer, in particular, considers the costs that he will eventually incur when he wants to sell, and so on for all future owners.

We build a dynamic asset-pricing model that captures these features. We study allocations, prices between investors, and marketmakers' bid and ask prices. We show how these equlibrium properties depend on investors' search abilities, marketmaker accessibility, and bargaining powers. We determine the search intensities that marketmakers choose, and derive the associated welfare implications of investment in marketmaking. Further, we show how search frictions may prevent information from being revealed through trading.

Our model of search is a variant of the coconuts model of Diamond (1982). A continuum of investors contact each other, independently, at some mean intensity λ, a parameter reflecting search ability. Similarly, marketmakers contact agents at some intensity ρ, a parameter reflecting dealer availability. When agents meet they bargain over the terms of trade. Gains from trade arise from heterogeneous discount rates, or from heterogeneous costs or benefits of holding assets. For example, an agent with a high discount rate or with costs of holding assets may be viewed as one in financial distress. The search-and-bargaining structure of our trading model is similar to that of the monetary model of Trejos and Wright (1995); our objectives and results are different.

Some of the research on the impact of transactions costs on asset pricing, for example by Amihud and Mendelson (1986), Constantinides (1986), Vayanos (1998), and Huang (1998), concentrate on exogenously specified trading costs. The endogenous impact of asymmetric information on trading costs and asset prices has been addressed by Wang (1993) and Gârleanu and Pedersen (2000). We complement this literature with an analytically tractable framework for asset pricing in the presence of search and bargaining.

While abstract, we view the theory of asset pricing, brokerage activity, and spreads presented here as helpful and empirically relevant (although by no means complete) for many off-exchange bilateral-trade markets in which one does not anticipate immediate identification of counterparties with whom there are likely gains from trade. These may include certain over-the-counter (OTC) markets such as those for mortgage-backed securities, corporate bonds, emerging-market debt, bank loans, and certain OTC derivatives, among others.

Although we do not address some of the salient features of real-estate markets, especially heterogeneous preferences over multi-dimensional asset quality, we believe that we do capture some of the impact on real-estate values of the roles of search and bargaining, the relative impatience of investors for liquidity and of their outside options for trade, and the role and profitability of brokers.

Duffie, Gârleanu, and Pedersen (2001) use this modeling framework to characterize the impact on asset prices and securities lending fees of the common institution by which would-be shortsellers must locate lenders of securities before being able to sell short. Difficulties in locating lenders of shares can allow for dramatic price "imperfections," as, for example, in the case of the spinoff of Palm, Incorporated, documented by Lamont and Thaler (2001).

We show that our model specializes in a specific way to the standard general-equilibrium paradigm as bilateral trade becomes increasingly active, under conditions to be described, extending a chain of results by Rubinstein and Wolinsky (1985), Gale (1987), Gale (1986a), Gale (1986b), and McLennan and Sonnenschein (1991), in a manner explained later in our paper. Thus, "standard" asset-pricing theory is not excluded, but rather is found at the end of the spectrum of increasingly "active" markets.

Market frictions have been used to explain the existence and behavior of marketmakers. For example, marketmakers' bid and ask prices have been explained by inventory considerations (Garman (1976), Amihud and Mendelson (1980), and Ho and Stoll (1981)), and by adverse selection arising from asymmetric information (Bagehot (1971), Glosten and Milgrom (1985), and Kyle (1985)). In our model, bid and ask prices are set in light of search frictions. We consider differences between the behavior of monopolistic and of competing marketmakers. Gehrig (1993) and Yavaş (1996) consider monopolistic marketmaking in one-period models in which investors may search for each other. We find, however, that the dynamics of our setting are important in determining agents' bargaining positions, and thus asset prices, bid-ask spreads, and investments in market-making capacity.

Marketmakers' bid and ask prices depend on investors' reservation values, which reflect the accessibility of marketmakers as well as investors' own abilities to find counterparties. Indeed, we show that marketmakers' bid-ask spreads approach zero as investors' search frictions become negligible. The experimental results of Lamoureux and Schnitzlein (1997) support this intuition. Gehrig (1993) and Yavaş (1996) have already shown in one-period models that bid-ask spreads decline with increasing investor search.

In our model, despite the bilateral nature of bargaining between a marketmaker and an investor, marketmakers are effectively in competition with each other over order flow, given the option of investors to search for better terms. Thus, Walrasian equilibria obtain in the limit as marketmakers' contact intensities become large, provided that marketmakers do not have total bargaining power. In contrast, an increase in the search intensity of a monopolistic marketmaker actually leads to wider spreads, due to the worsening of the investors' outside options, through a reduction in the number of agents with whom there are trading gains.

Studying endogenous search in labor markets, Mortensen (1982) and Hosios (1990) find that agents may choose inefficient search levels because they do not internalize the gains from trade realized by future trading partners.[1] We consider marketmakers' choices of search intensity, and the social efficiency of these choices. A monopolistic marketmaker imposes a negative externality on investors because his intermediation renders less valuable the opportunity of investors to trade directly with each other. A monopolistic marketmaker thus provides a higher than socially efficient level of intermediation. Competitive marketmakers may provide even more intermediation, as they do not take into account how their search intensities affect the equilibrium allocation of assets among investors.

If investors may have asymmetric information about future dividends and can observe prices, then prices may reveal all or part of the private information through a rational-expectations equilibrium (Grossman (1981) and Grossman and Stiglitz (1980)). In a setting related to ours, with bargaining and asymmetric information, Wolinsky (1990) constructs a steady-state partially-revealing equlibrium.[2] Introducing asymmetric information, we provide a simple example in which investors are sufficiently anxious to arrive at a bargain that they trade at "pooling prices" that reveal no information at all.

[1] Moen (1997) shows that search markets can be efficient under certain conditions.

[2] See also Serrano and Yosha (1993) and Serrano and Yosha (1996).

Potential extensions of our model might allow for endogenous fluctuation in the population of, and ease of identification of, active buyers, due to feedback effects between asset returns and the creditworthiness of investors.

2 Trade among Investors

This section introduces an economy in which agents can trade only when they meet each other. Transaction prices are determined through bargaining. We compare allocations and prices to those prevailing in a perfect Walrasian market. Later, in Section 3, we introduce marketmakers.

2.1 Model

We fix a probability space $(\Omega, \mathcal{F}, Pr)$ and a filtration $\{\mathcal{F}_t : t \geq 0\}$ of sub-σ-algebras satisfying the usual conditions, as defined by Protter (1990), representing the resolution over time of information commonly available to investors. (Asymmetric information is briefly considered later in the paper.)

A single non-storable consumption good is used as a numeraire. A single asset pays a strictly positive progressively-measurable dividend process X.

For simplicity, we suppose a constant conditional expected dividend growth rate of c, so that, for any times t and $s > t$, we have $E_t(X_s) = X_t e^{c(s-t)}$, where E_t denotes expectation conditional on the information $\mathcal{F}_t$ available at time t. This includes such traditional examples as geometric Brownian dividends, or a consol bond, for which $X_t = 1$ for all t.

Each agent is risk-neutral and infinitely lived, with a time-preference type, "high" or "low," described by a two-state Markov chain. A high-type agent has a high rate r_h of time preference.[3] A low-type agent has a time preference rate $r_l \leq r_h$. The switching intensity of low to high is λ_u; the switching intensity from high to low is λ_d.

In order to illustrate certain concepts, we also suppose that high-type agents lose a fraction $\delta \geq 0$ of any asset cash flows. Of the inequalities

[3] A discount-rate process r is predictable, with $\int_0^T |r(t)|\, dt < \infty$ almost surely. A cumulative consumption process is a finite-variation process C with the property that $E\left[\int_0^\infty \exp\left(\int_0^t -r(s)\, ds\right) \, (dC^+(s) + dC^-(s))\right] < \infty$, where C can be decomposed as $C = C^+ - C^-$, with C^+ and C^- increasing adapted processes. Consumption processes are ranked by an agent with discount rate r according to the utility function that assigns to each cumulative consumption process C the utility $E\left[\int_0^\infty \exp\left(\int_0^t -r(s)\, ds\right) \, dC(t)\right]$.

$r_l \leq r_h$ and $\delta \geq 0$, at least one is strict, setting up strict gains from trade. For a transversality-like condition, we assume that $c < r_l$.

A fraction s of investors are initially endowed with one unit of the asset. Investors can hold at most one unit of the asset and cannot shortsell. Because agents have linear utility, we can restrict attention to equilibria in which, at any given time and state of the world, an agent holds either 0 or 1 unit of the asset. Hence, the full set of agent types is $\mathcal{T} = \{ho, hn, lo, ln\}$, with the letters "$h$" and "$l$" designating the agent's time-preference state, as above, and with "o" or "n" indicating whether the agent owns the asset or not, respectively.

We suppose that there is a "continuum" (a non-atomic finite measure space) of agents, and let $\mu_\sigma(t)$ denote the fraction at time t of agents of type $\sigma \in \mathcal{T}$. Agent's time-preference type processes are pair-wise independent, setting up a later application of the law of large numbers.

Because the fractions of each type of agent add to 1 at any time t,

$$\mu_{ho}(t) + \mu_{hn}(t) + \mu_{lo}(t) + \mu_{ln}(t) = 1. \tag{1}$$

Because the total fraction of agents owning an asset is s,

$$\mu_{ho}(t) + \mu_{lo}(t) = s. \tag{2}$$

Any two agents are free to trade the asset whenever they meet, for a mutually agreeable number of units of current consumption. (The determination of the terms of trade is to be addressed later.) Agents meet, however, only at random times, in a manner idealized as follows. At the event times of a Poisson process with some intensity parameter λ, an agent contacts some other agent, chosen at random. The exponential inter-contact-time distribution is natural, as it would arise from Bernoulli (independent success-failure) trials at contact, with a probability of $\lambda\Delta$ of successful contact during a contact-time interval of length Δ, in the limit as Δ goes to zero.

Because, conditional on a contact, the agent chosen for contact is drawn at random, "equally likely," the probability of contacting an agent who is a member of a set of agents of mass $\overline{\mu}$ is $\overline{\mu}$. We suppose that the contact processes of agents are pair-wise independent, and appeal informally to the law of large numbers (see Footnote 7), under which, for a set of agents of current mass $\mu_A(t)$, contact is made with another group of agents of current mass $\mu_B(t)$ continually at the total current rate $2\lambda\mu_A(t)\mu_B(t)$. Our random-matching formulation and appeal to the law of large numbers is typical of the recent monetary literature (for instance, Trejos and Wright (1995) and references therein).[4] We also suppose that random switches in time-preference types are independent of the matching processes.

An alternative to our informal appeal to the law of large numbers is to construct a sequence of random-matching economies with increasingly large finite populations, and to treat our results in the form of limits of equilibria, which seems an unappealing distraction from our main goal.

2.2 Dynamic Bargaining Equilibrium

In this section we compute explicitly the allocations and prices in a dynamic bargaining equilibrium.

In equilibrium, trades are between high-type (impatient to consume) asset owners and low-type non-owners. When these agents meet, they bargain over the price. An agent's bargaining position depends on his outside option, which in turn depends on the mass of other counterparties, both now and in the future. In deriving the equilibrium, we rely on the insight from bargaining theory that trade happens instantly.[5] This allows us to derive a dynamic bargaining equilibrium in two steps. First, we derive the equilibrium masses of the different investor types. Second, we compute agents' value functions and transaction prices (taking as given the masses).

The rate of change of the mass $\mu_{ho}(t)$ of high-type owners is

$$\dot{\mu}_{ho}(t) = -2\lambda\mu_{ln}(t)\mu_{ho}(t) - \lambda_d\mu_{ho}(t) + \lambda_u\mu_{lo}(t). \tag{3}$$

The first term reflects the fact that agents of type ln contact those of type ho at a total rate of $\lambda\mu_{ln}(t)\mu_{ho}(t)$, while agents of type ho contact those of type ln at the same total rate $\lambda\mu_{ln}(t)\mu_{ho}(t)$. At both of these types of encounters, the agent of type ho becomes one of type hn. This implies a total rate of reduction of mass due to these encounters of $2\lambda\mu_{ln}(t)\mu_{ho}(t)$. The last two

[4]More generally, if search intensities vary across agents, with agent x having contact intensity $\lambda(x)$, then, under regularity conditions for random matching, contact between a subset A of agents and a subset B of agents would occur continually at the total rate $\mu(B)\int_A \lambda(x)\,\mu(dx) + \mu(A)\int_B \lambda(x)\,\mu(dx)$, where μ is the measure on the space of agents.

[5]In general, bargaining leads to instant trade when agents do not have asymmetric information. Otherwise there can be strategic delay. In our model, it does not matter whether agents have private information about their own type for it is common knowledge that a gain from trade arises only between between high-type asset owners and low-type non-owners.

terms reflect the migration of owners from high to low discount rates, and from low to high discount rates, respectively.

The rate of change of μ_{ln} is, likewise,

$$\dot{\mu}_{ln}(t) = -2\lambda\mu_{ln}(t)\mu_{ho}(t) - \lambda_u\mu_{ln}(t) + \lambda_d\mu_{hn}(t). \tag{4}$$

When agents of type ln and ho trade, they become of type lo and hn, respectively, so

$$\dot{\mu}_{lo}(t) = 2\lambda\mu_{ln}(t)\mu_{ho}(t) - \lambda_u\mu_{lo}(t) + \lambda_d\mu_{ho}(t) \tag{5}$$

and

$$\dot{\mu}_{hn}(t) = 2\lambda\mu_{ln}(t)\mu_{ho}(t) - \lambda_d\mu_{hn}(t) + \lambda_u\mu_{ln}(t). \tag{6}$$

We note that Equations (1)–(4) imply Equations (5)–(6).

In most of the paper we focus on stationary equilibria, that is, equilibria in which the masses are constant. In our welfare analysis, however, it is more natural to take the initial masses as given, and, therefore, we develop some results with any initial mass distribution. The following proposition asserts the existence, uniqueness, and stability of the steady state.

Proposition 1 *There is a unique constant solution* $\mu = (\mu_{ho}, \mu_{hn}, \mu_{lo}, \mu_{ln}) \in [0,1]^4$ *to equations (1)–(6). From any initial condition* $\mu(0) \in [0,1]^4$ *satisfying (1) and (2), the unique solution* $\mu(t)$ *to this system of equations converges to* μ *as* $t \to \infty$.

A particular agent's type process $\{\sigma_t : -\infty < t < +\infty\}$ is, in steady-state, a 4-state Markov chain with state-space $\mathcal{T}$, and with constant switching intensities determined in the obvious way[6] by the steady-state population masses μ and the intensities λ, λ_u, and λ_d, and with a steady-state probability distribution that is the same as the equilibrium constant cross-sectional distribution μ of types characterized in Proposition 1.[7]

[6] For example, the transition intensity from state ho to state lo is λ_d, the transition intensity from state ho to state hn is $2\lambda\mu_{hn}$, and so on, for the 4×3 switching intensities.

[7] Intuitively, this follows from the law of large numbers. Formally, we use Theorem C of Sun (2000) to construct our probability space $(\Omega, \mathcal{F}, Pr)$ and agent space $[0,1]$, with an appropriate σ-algebra making $\Omega \times [0,1]$ into what Sun calls a "rich space," with the properties that: *(i)* for each individual agent in $[0,1]$, the agent's type process is indeed a Markov chain in $\mathcal{T}$ with the specified generator, *(ii)* the unconditional probability

We now turn to the determination of transaction prices. We first conjecture, and verify shortly, a natural steady-state equilibrium utility at time t for remaining lifetime consumption for a particular agent that depends only on the agent's current type $\sigma_t \in \mathcal{T}$ and the current dividend rate X_t, so that we may write $V(X_t, \sigma_t)$ for this utility. Likewise, we conjecture that the trade price at time t is of the form $P(X_t)$ for some $P : \mathbb{R}_+ \to \mathbb{R}_+$.

In order to calculate V and P, we consider a particular agent and a particular time t, let τ_r denote the next (stopping) time at which that agent's time-preference type changes, let τ_m denote the next (stopping) time at which a counterparty with gain from trade is met, and let $\tau = \min\{\tau_r, \tau_m\}$. Then, by definition,

$$\begin{aligned}
V(X_t, ho) &= E_t\Big[\int_t^{\tau} e^{-r_h(u-t)}(1-\delta)X_u\,du + e^{-r_h(\tau_r-t)}V(X_{\tau_r}, lo)1_{\{\tau_r<\tau_m\}} \\
&\quad + e^{-r_h(\tau_m-t)}\left(V(X_{\tau_m}, hn) + P(X_{\tau_m})\right)1_{\{\tau_r\geq\tau_m\}}\Big] \\
V(X_t, hn) &= E_t\left[e^{-r_h(\tau_r-t)}V(X_{\tau_r}, ln)\right] \\
V(X_t, lo) &= E_t\left[\int_t^{\tau_r} e^{-r_l(u-t)}X_u\,du + e^{-r_l(\tau_r-t)}V(X_{\tau_r}, ho)\right] \\
V(X_t, ln) &= E_t\Big[e^{-r_l(\tau_r-t)}V(X_{\tau_r}, hn)1_{\{\tau_r<\tau_m\}} + \\
&\quad e^{-r_l(\tau_m-t)}\left(V(X_{\tau_m}, lo) - P(X_{\tau_m})\right)1_{\{\tau_r\geq\tau_m\}}\Big].
\end{aligned} \tag{7}$$

A low-type non-owner has a reservation value $\Delta V_l(X_t) = V(X_t, lo) - V(X_t, ln)$ for buying the asset, and a high-type owner has a reservation value $\Delta V_h(X_t) = V(X_t, ho) - V(X_t, hn)$ for selling the asset. The gain from trade between these agents is $\Delta V_l(X_t) - \Delta V_h(X_t)$. We study equilibria in which the seller gets a fixed fraction, q, of the gain from trade, in that

$$P(X_t) \;=\; \Delta V_h(X_t)(1-q) + \Delta V_l(X_t)q\,. \tag{8}$$

This means that the seller's bargaining power is q. Not all models of bargaining allow the equilibrium bargaining outcome to depend on agents' outside

distribution of the agents' type is always the steady-state distribution μ on $\mathcal{T}$ given by Proposition 1, *(iii)* agents' type transitions are almost everywhere pair-wise independent, and *(iv)* the cross-sectional distribution of types is also given by μ, almost surely, at each time t. This result settles the issue of existence of the proposed equilibrium joint probabilistic behavior of individual agent type processes with the proposed cross-sectional distribution of types. This still leaves open, however, the existence of a random-matching process supporting the proposed type processes.

options, as we do.[8] We show in Section 4.3, however, that (8) is the outcome of an alternating-offers bargaining game, and compute q as an explicit function of the model parameters. With a fixed q, (8) is the outcome of Nash (1950) bargaining, and any q can be justified in equilibrium by the simultaneous-offer bargaining game described in Kreps (1990).

Because of the assumption that X has a constant expected growth rate and the fact that the stopping times considered are the first jump times of counting processes with constant intensities, there exists an equilibrium in which the value functions and prices are proportional to X. That is, there is an equilibrium with $V(X_t, \sigma) = v_\sigma X_t$ and $P(X_t) = pX_t$, for coefficients $(v_{ho}, v_{hn}, v_{lo}, v_{ln}, p)$. With this, (7)–(8) imply the following equations for the steady-state equilibrium coefficients.

Theorem 2 *There is a steady-state subgame-perfect Nash equilibrium in which the value and price coefficients* $(v_{ho}, v_{hn}, v_{lo}, v_{ln}, p)$ *uniquely solve*

$$\begin{aligned}
0 &= r_h v_{ho} - \lambda_d(v_{lo} - v_{ho}) - 2\lambda\mu_{ln}(p - v_{ho} + v_{hn}) - (1-\delta) \\
0 &= r_h v_{hn} - \lambda_d(v_{ln} - v_{hn}) \\
0 &= r_l v_{lo} + \lambda_u(v_{lo} - v_{ho}) - 1 \\
0 &= r_l v_{ln} + \lambda_u(v_{ln} - v_{hn}) + 2\lambda\mu_{ln}(p - v_{lo} + v_{ln}) \\
p &= (v_{ho} - v_{hn})(1-q) + (v_{lo} - v_{ln})q.
\end{aligned} \tag{9}$$

These equations have a unique solution because the associated coefficient matrix is non-singular. A dynamic-programming argument found in the appendix confirms that the proposed investor strategies constitute an (infinite-agent, infinite-time) subgame-perfect Nash equilibrium. That is, if two agents with gains from trade meet at time t, the potential buyer tenders the price $P(X_t)$, the potential seller tenders the same price $P(X_t)$, and both prefer to immediately trade at that commonly announced price. The property that $r_l > c$ is used for "transversality."

More generally, if we allow for non-steady state equilibria (determined by initial masses $\mu(0)$ of agent types that are not at steady state), we may

[8] Intuitively, outside options do matter here because there is a risk of a breakdown of bargaining due to changes in discount rates (Binmore, Rubinstein, and Wolinsky (1986)), and because the value stems in part from dividends paid during bargaining. The matter is complicated, however, by the complex nature of the outside option, which is given by several factors: change in discount rate, meeting another trading partner, and dividends.

treat the coefficients $(v_{ho}, v_{hn}, v_{lo}, v_{ln}, p)$ as time-dependent, and obtain the differential equations

$$\begin{aligned}
\dot{v}_{ho} &= r_h v_{ho} - \lambda_d(v_{lo} - v_{ho}) - 2\lambda\mu_{ln}(p - v_{ho} + v_{hn}) - (1-\delta) \\
\dot{v}_{hn} &= r_h v_{hn} - \lambda_d(v_{ln} - v_{hn}) \\
\dot{v}_{lo} &= r_l v_{lo} + \lambda_u(v_{lo} - v_{ho}) - 1 \\
\dot{v}_{ln} &= r_l v_{ln} + \lambda_u(v_{ln} - v_{hn}) + 2\lambda\mu_{ln}(p - v_{lo} + v_{ln}) \\
p &= (v_{ho} - v_{hn})(1-q) + (v_{lo} - v_{ln})q,
\end{aligned} \tag{10}$$

suppressing from the notation the dependence of $(v(t), p(t), \mu(t))$ on t. For a given q, coupled with (1)-(6), these equations have a unique solution that satisfies the natural boundary condition: $\lim_{t\to\infty} e^{-r_l t} v(t) = 0$.

The game and equilibrium could be modified to allow for an exogenous fractional loss of price at each trade as an administrative transactions cost, with solutions of a similar linear form. We could also allow the discount rates, r_l and r_h, to be themselves Markov chains, and get a richer class of linear equilibria in which there are "regimes" for prices.

2.3 Walras Equilibrium

The allocation associated with the equilibrium treated in Theorem 1 is efficient among all mechanisms that re-allocate the asset, pair-wise, at contact times, but is obviously not efficient among all mechanisms that can allocate at any time to any agents. The Walrasian competitive market equilibrium allocation is efficient in this stronger sense. A Walrasian equilibrium is characterized by a single price process at which agents may buy and sell *instantly*, such that supply equals demand at each state and time. In a Walrasian allocation, because it is efficient, all assets are held by agents with a low discount rate, if there are enough such agents, which is the following condition.

Condition 1 $s < \lambda_d/(\lambda_u + \lambda_d)$.

Our results, however, apply generally.

Under Condition 1, the unique Walras equilibrium has agent masses

$$\begin{aligned}
\mu_{lo}^* &= s \\
\mu_{ln}^* &= \frac{\lambda_d}{\lambda_u+\lambda_d} - s \\
\mu_{ho}^* &= 0 \\
\mu_{hn}^* &= \frac{\lambda_u}{\lambda_u+\lambda_d}.
\end{aligned} \tag{11}$$

The Walrasian price is

$$P_t^* = E_t\left[\int_0^\infty e^{-r_l s} X_{t+s}\, ds\right] = p^* X_t,$$

where $p^* = (r_l - c)^{-1}$. The Walras equilibrium price, a version of what is sometimes called the "Gordon dividend growth model" of valuation, is the value of holding the asset forever for a hypothetical agent who always has a low discount rate.

If Condition 1 is not satisfied, the marginal investor has a high discount rate, and the Walrasian price is the expected value of holding the asset indefinitely for a (hypothetical) agent who always has a high discount rate. In this case $\mu_{ln}^* = 0$, and the other masses are determined in the obvious way.

The Walrasian equilibrium is approached by bargaining equilibria as agents meet increasingly frequently in the following sense.

Theorem 3 *Suppose that either $q > 0$ and Condition 1 applies, or that $q < 1$ and Condition 1 does not apply. Let $\lambda^k \to \infty$, and let (μ^k, p^k) be the corresponding sequence of stationary bargaining equilibria. Then $(\mu^k, p^k) \to (\mu^*, p^*)$.*

The condition on q amounts to requiring that agents on the "short" side of the market (under Condition 1, those with high discount rates) have some bargaining power. Otherwise, agents on the other side have all of the bargaining power and cannot be kept indifferent between trading and not trading.

Contrary to our result, Rubinstein and Wolinsky (1985) find, in a model similar in spirit to ours, that the bargaining equilibrium does *not* converge to the competitive equilibrium as trading frictions approach zero. In their model, however, agents disappear after they trade, and new agents enter the economy such that the masses, which are exogenous, of buyers and sellers stay constant.[9] Gale (1987) argues that this failure is due to the fact that the total mass of agents entering their economy is infinite, which makes the competitive equilibrium of the total economy undefined. Gale (1987) shows that if the total mass of agents is finite, then the economy (which is not stationary) is Walrasian in the limit. He suggests that, when considering stationary economies, one should compare the bargaining prices to those of a "flow equilibrium" rather than a "stock equilibrium." Our model has a natural determination of steady-state masses, even though no agent enters the economy. This is accomplished by letting agents switch types randomly. Other important differences between our framework and that of Rubinstein and Wolinsky (1985) are that we accommodate repeated trade, and that we diminish search frictions explicitly through λ rather than implicitly through the discount rate. See Bester (1988, 1989) for the importance of dimishing search frictions directly.

We are able to reconcile a steady-state economy with convergence to Walrasian outcomes in both a flow and stock sense, and both for allocations and for prices. In Section 3.1, we shall see whether a Walrasian equilibrium can also be approached by increasing the amount of intermediation offered by broker-dealers.

2.4 Numerical Example

We consider an illustrative example. Table 1 contains the exogenous parameters, Table 2 contains the implied stationary masses, and Table 3 contains the steady-state coefficients for the value functions and prices. For these parameters, agents contact other agents at an expected rate of more than once per week ($\lambda = 60$), have a "normal" discount rate of $r_l = 5\%$, and are in financial distress, with a high discount rate of $r_h = 25\%$, 1 year out of every 11 years, on average. An agent of type ho has a fraction $q_h = 0.499$ of the bargaining power when bargaining with an agent of type ln. (This bargaining power is found endogenously using the explicit bargaining model of Section 4.3.) At any time, $s = 20\%$ of the agents have the asset.

Table 2 shows that almost all of the assets are held by agents with a

[9] Binmore and Herrero (1988) consider a similar model, in which they vary the mass of agents that enters the economy. They find that prices do converge to competitive prices when there is no entry. Gale (1986a), Gale (1986b), and McLennan and Sonnenschein (1991) show that a bargaining game implements Walrasian outcomes in the limiting case with no frictions (that is, no discounting) in much richer settings for preferences and goods.

λ	λ_u	λ_d	s	r_h	r_l	c	q	δ
60.00	0.10	1.00	0.20	0.25	0.05	0.03	0.499	0

Table 1: Base-case parameters.

μ_{ho}	μ_{hn}	μ_{lo}	μ_{ln}
0.0002	0.0907	0.1998	0.7093

Table 2: Steady-state masses corresponding to base-case parameters.

low discount rate; only about 1 unit per thousand of the asset is held by agents with a high discount rate. That is, the allocation is "nearly efficient." Table 3 shows, however, that the price is discounted by almost 3% from the Walrasian price, which has a price-dividend ratio of $(0.05 - 0.03)^{-1} = 50$.

Figure 1 shows how prices increase with the contact intensity, λ, holding other base-case parameters fixed. (We hold bargaining powers fixed as we vary λ. Regarding this point, see Section 4.3.) We see that, as agents meet more easily, allocations become more efficient and bargaining becomes "less fierce," given the outside option of quickly finding other trading partners.

A steady-state fraction of 10/11 of agents have the low discount rate, explaining the big drop in prices shown in Figure 2 as s becomes close to this fraction.

Figure 3 shows that prices are increasing in the seller's bargaining power. Figure 4 confirms the intuition that an increase in the severity of a personal liquidity shock drives down the price, although the Walrasian price is unaffected.

v_{ho}	v_{hn}	v_{lo}	v_{ln}	p
48.57	0.07	48.81	0.08	48.62

Table 3: Base-case coefficients for value functions and prices.

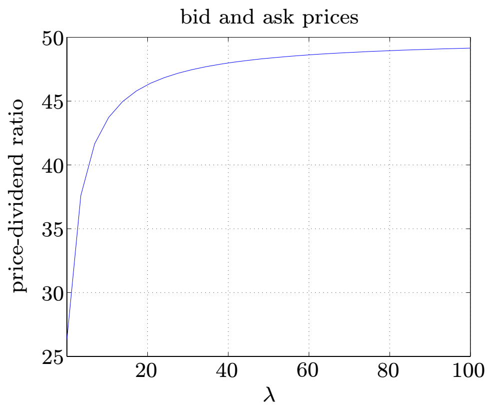

Figure 1: Dependence of the price-dividend ratio, p, on the search intensity, λ.

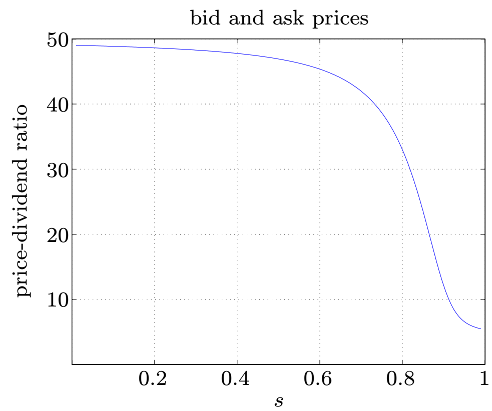

Figure 2: Dependence of the price-dividend ratio, p, on the total asset supply, s.

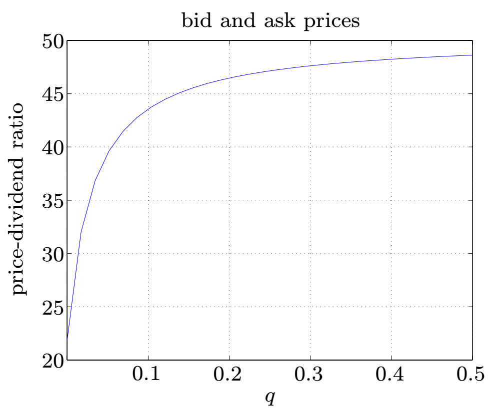

Figure 3: Dependence of the price-dividend ratio, p, on the seller's bargaining power, q.

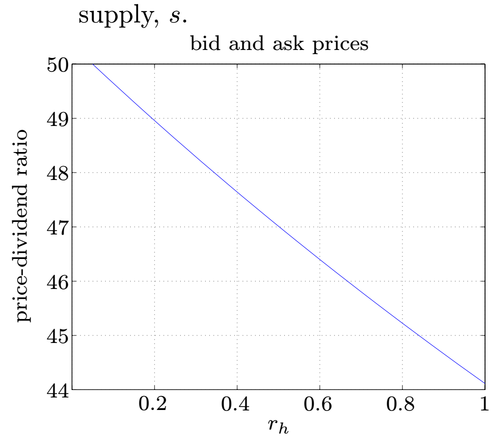

Figure 4: Dependence of the price-dividend ratio, p, on the magnitude of the high discount rate, r_h.

3 Marketmakers

This section introduces marketmakers, studying bid prices, ask prices, and prices negotiated directly between investors. We focus on the implications of search and bargaining, abstracting from other issues that affect marketmaking behavior, such as asymmetric information, risk-aversion, and inventory management. Section 3.1 considers a monopolistic marketmaker. Section 3.2

addresses competing marketmakers.

3.1 Monopolistic Market Making

We suppose that investors can trade with the marketmaker only when they meet one of the marketmaker's non-atomic "dealers." We assume that there is a unit mass of such dealers who contact potential investors randomly and pair-wise independently, letting ρ be the intensity with which a dealer contacts a given agent. It would be equivalent to have a mass k of dealers with contact intensity ρ/k, for any $k > 0$.

Dealers instantly balance their positions with their market-making firm, which, as a whole, does not hold inventory. When an investor meets a dealer, the dealer is assumed to have all of the bargaining power, and quotes an ask price, aX, and a bid price, bX, that are, respectively, a buyer's and a seller's reservation value. A marketmaker with all bargaining power affects investors' value functions only through his effect on equilibrium masses. Hence, given the equilibrium masses, these value functions can be computed from (9). Pairs of investors with gains from trade, when they meet, trade at a bargained price of pX. In equilibrium, $b \leq p \leq a$.

We now derive the equilibrium masses in the presence of the dealers. In the case we examine, there are more agents willing to own the asset at the dealer-market price than there are assets to be shared (the converse obtains in the complementary case). Rationing will thus occur, but agents are indifferent to being rationed, as monopolistic dealers quote their reservation prices for trade. Specifically, because the steady-state fraction of low-type agents is $\lambda_d(\lambda_u + \lambda_d)^{-1}$, we have

$$\mu_{ln} = \frac{\lambda_d}{\lambda_u + \lambda_d} - s + \mu_{ho}.$$

Under Condition 1, this implies that the total contact rate, $\rho\mu_{ln}$, of dealers with potential buyers is strictly larger than the total contact rate, $\rho\mu_{ho}$, of dealers with potential sellers. As a result, all potential sellers trade when in contact with dealers, while potential buyers are rationed by dealers. (To settle the issue, one can assume random rationing.) Analogously, when Condition 1 is not satisfied, the sell side is rationed.

The equilibrium is calculated as before, replacing the steady-state equilibrium masses with the constant solution to (1), (2), and

$$\dot{\mu}_{ho}(t) = -\left(2\lambda\mu_{ln}(t)\mu_{ho}(t) + \rho\mu_m(t)\right) - \lambda_d\mu_{ho}(t) + \lambda_u\mu_{lo}(t) \tag{12}$$
$$\dot{\mu}_{ln}(t) = -\left(2\lambda\mu_{ln}(t)\mu_{ho}(t) + \rho\mu_m(t)\right) + \lambda_d\mu_{hn}(t) - \lambda_u\mu_{ln}(t) \tag{13}$$
$$\dot{\mu}_{lo}(t) = \left(2\lambda\mu_{ln}(t)\mu_{ho}(t) + \rho\mu_m(t)\right) + \lambda_d\mu_{ho}(t) - \lambda_u\mu_{lo}(t) \tag{14}$$
$$\dot{\mu}_{hn}(t) = \left(2\lambda\mu_{ln}(t)\mu_{ho}(t) + \rho\mu_m(t)\right) - \lambda_d\mu_{hn}(t) + \lambda_u\mu_{ln}(t), \tag{15}$$

where $\mu_m(t) = \min\{\mu_{ho}(t), \mu_{ln}(t)\}$.[10] The first terms in (12)–(15) reflect the total rates of trade, both directly between investors and through dealers.

Proposition 4 *There is a unique constant solution $\mu = (\mu_{ho}, \mu_{hn}, \mu_{lo}, \mu_{ln}) \in [0,1]^4$ to (1), (2), and (12)-(15). From any initial condition $\mu(0) \in [0,1]^4$ satisfying (1) and (2), the unique solution $\mu(t)$ to this system of equations converges to μ as $t \to \infty$.*

We now consider the effect of an increasingly large intensity ρ of marketmaking.

Theorem 5 *Let (ρ^k) be an increasing sequence of positive real numbers converging to ∞. The corresponding sequence (μ^k, b^k, a^k, p^k) of unique stationary bargaining equilibria converges, and the bid-ask spread, $a^k - b^k$, is increasing.*

We note that the limit equilibrium coincides with the equilibrium in an economy in which the marketmaker can be approached instantly.

The bid-ask spread widens with increases in the dealer contact intensity ρ because an investor's potential "threat" to search for a direct trade with another investor becomes increasingly less persuasive, since the mass of investors with whom there are gains from trade shrinks.

We have seen that the existence of an effective (large-ρ) monopolistic marketmaker leads to efficient allocations and a large profit earned by the marketmaker. A natural question is whether a monopolistic marketmaker can sustain this large profit in an economy in which investors have little need for intermediation, that is, when λ is high. This question is not trivial because, for any finite λ, *all* trades are made using the marketmaker, provided the marketmaker can be approached instantly ($\rho = +\infty$). The following theorem shows that the marketmaker's profit indeed vanishes when investors' potential for bilateral trade increases, regardless of the nature of intermediation.

[10] The minimum operator is used to determine whether the buy or sell side is rationed.

Theorem 6 *Let (λ^k) be a sequence of positive real numbers such that $\lambda^k \to \infty$, (ρ^k) be a sequence in $[0, \infty]$, and let (μ^k, b^k, a^k, p^k) be the corresponding sequence of stationary bargaining equilibria with a monopolistic marketmaker. Then b^k, a^k, and p^k converge to the Walrasian price coefficient p^*.*

This highlights the importance of considering the option of investors to search for direct trades among themselves, even though this option may not be taken in equilibrium.

3.2 Competing Marketmakers

We now turn to the case of competing marketmakers. If marketmakers can be approached instantly by investors, the Walrasian outcome obtains with two or more marketmakers playing a Bertrand game.

The case in which marketmakers cannot be approached instantly is more interesting, and captures the idea that an investor must bargain with each marketmaker sequentially. We assume that there is a unit mass of independent non-atomic marketmakers with a fixed intensity, ρ, of meeting an investor. To avoid considering marketmaker inventory, we assume that there is an inter-dealer market in which marketmakers can buy and sell instantly at price mX, and that marketmakers do not hold inventory. Each marketmaker has a bid price, bX, and an ask price, aX. As opposed to the monopolistic case, we assume that marketmakers have a fraction, $z \in [0, 1]$, of the bargaining power when facing an investor.

An equilibrium under Condition 1 is as follows. (If Condition 1 fails, the result is analogous.) The steady-state equilibrium investor masses, μ, are found using (12)–(15), as for a monopolistic marketmaker. The investors' value functions are modified for marketmakers with limited bargaining power. This computation is analogous to that of the basic model, and is outlined in the appendix. The inter-dealer price, mX, is equal to the ask price, aX, and to any buyer's reservation value, $(v_{lo} - v_{ln})X$, since both dealers and buyers must be indifferent between trading with each other and not trading. The bid price is bX, where $b = (1 - z)m + z(v_{ho} - v_{hn})$, reflecting the power of marketmakers to extract a fraction z of the difference between the interdealer market price and a seller's reservation value.

If investors have all of the bargaining power (that is, $z = 0$), the bid-ask spread is zero at all times, and the equilibrium approaches the Walrasian equilibrium, in both prices and allocations, as marketmaking becomes more intense (that is, for increasing ρ). This situation can be interpreted as one in which investors meet different marketmakers at the same time.

On the other hand, if marketmarkers have all of the bargaining power (that is, $z = 1$), the equilibrium is the same as the equilibrium with a monopolistic marketmaker. It might seem surprising that having many "competing" non-atomic marketmakers is equivalent to having a monopolistic marketmaker. The result follows from the fact that a search economy is inherently un-competitive, in that each time agents meet a bilateral bargaining relationship obtains. We emphasize that the monopolistic rents to "competing" dealers depend on the credibility of their bargaining power, and not on "collusion" among dealers.

For the natural intermediate case in which $z \in (0, 1)$, there is a strictly positive bid-ask spread, which is increasing in the marketmakers' bargaining power, z. As the level of intermediation increases ($\rho \to \infty$), the equilibrium approaches the Walrasian equilibrium. This, too, may seem surprising since an investor trades with the first marketmaker he meets, and this marketmaker could have almost all bargaining power (z close to 1). As ρ increases, however, the investor's outside option when bargaining with a marketmaker improves, because he can more easily meet another marketmaker. This results in a better price for the investor. This effect drives prices to their Walrasian levels as the intensity ρ approaches infinity.

The limit results stated in the previous paragraphs follow from Theorem 5 and from the next result.

Theorem 7 *Let (ρ^k) be a sequence of positive real numbers such that $\rho^k \to \infty$, and let (μ^k, b^k, a^k, p^k) be the corresponding sequence of stationary search equilibria with non-atomic (competing) marketmakers. Then $\mu^k \to \mu^*$. If $z < 1$, then b^k, a^k, and p^k converge to p^*.*

3.3 Numerical Example, Continued

We illustrate some of the effects of marketmaking discussed in this section by extending the example of Section 2.4. We consider the exogenous parameters of Table 1, as well as an intensity, $\rho = 100$, of agents meeting a marketmaker. Figure 5 shows how the investor price and the dealer's bid and ask prices depend on the bargaining power of the marketmaker. We see that all prices are decreasing in the marketmaker's bargaining power. Moreover, the bid-ask spread is increasing in the marketmaker's bargaining power.

Figure 6 shows how prices depend on the intensity, ρ, of meeting dealers in the cases of dealer bargaining power $z = 1$ and $z = 0.80$, respectively. Since allocations become more efficient as ρ increases, in both cases, all prices increase with ρ. Interestingly, the spreads are increasing with ρ in the case of $z = 1$, but decreasing in the case of $z = 0.80$. The intuition for this difference is as follows. When the dealers' contact intensity increases, they execute more trades. Investors then find it more difficult to contact other investors with whom to trade. If dealers have all of the bargaining power, this leads to wider spreads. If dealers don't have all of the bargaining power, however, then higher market-maker intensity leads to a narrowing of the spread because of any investor's improved threat of waiting to trade with the next marketmaker.

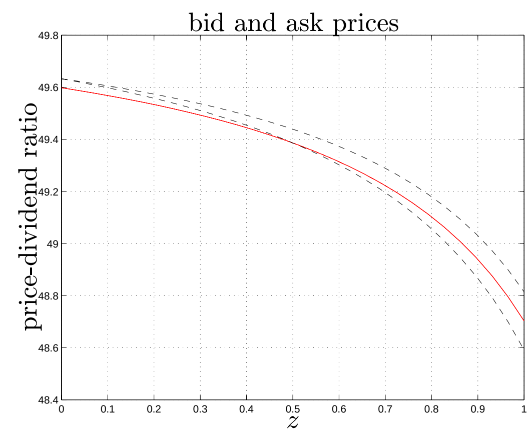

Figure 5: The solid line shows the price-dividend ratio at which investors trade with each other. The dashed lines show the bid (b) and ask (a) price coefficients used when investors trade with a marketmaker. The horizontal axis shows the bargaining power (z) of the marketmaker.

4 Analysis and Extensions

This section treats extensions. We endogenize the marketmakers' search intensities, discuss the welfare implications of the model, consider two explicit bargaining games, and extend the model so that investors have asymmetric information about the asset payoffs.

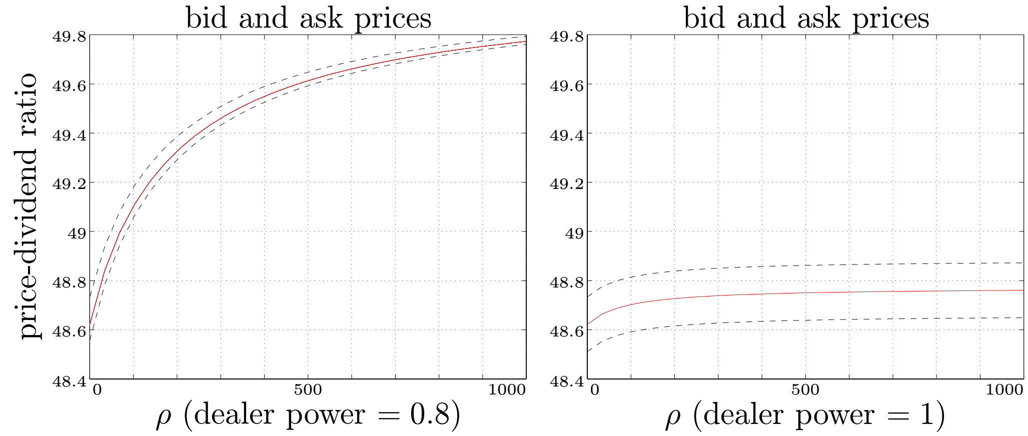

Figure 6: The solid line shows the price coefficient used when investors trade with each other. The dashed lines show the bid (b) and ask (a) price coefficients used when investors trade with a marketmaker. The prices are functions of the intensity (ρ) with which an investor meets a dealer. The bargaining power of the marketmaker is $z = 0.8$ in the left panel, and $z = 1$ in the right panel.

4.1 Endogenous Marketmaker Search

Here, we investigate the search intensities that marketmakers would optimally choose in the two cases considered above: a single monopolistic marketmaker and non-atomic competing marketmakers. We illustrate how marketmakers' choices of search intensities depend on: *(i)* the marketmakers' influence on the equilibrium allocations of assets, and *(ii)* the marketmakers' bargaining power. We take investors' search intensities as given. Considering the interactions arising if both investors and intermediaries choose search levels endogenously would be an interesting issue for future research.[11]

Because the marketmakers' search intensity affects the masses, μ, of investor types, it is natural to take as given the initial masses, $\mu(0)$, of investors, rather than to compare based on the different steady-state masses corresponding to different choices of search intensities. Hence, in this section, we are not relying on a steady-state analysis.

We assume that a marketmaker chooses one search intensity and abides by it. This assumption is convenient, and can be motivated by interpreting the search intensity as based on a technology that is difficult to change. A

[11]Relatedly, Pagano (1989) considers a one-period model in which investors choose between searching for a counterparty and trading on a centralized market.

full dynamic analysis of the optimal control of marketmaking intensities with small switching costs would be interesting, but seems difficult. We merely assume that marketmakers choose ρ so as to maximize the present value, using some discount rate that we denote r, of future marketmaking spreads, net of the rate $\Gamma(\rho)$ of technology costs, where $\Gamma : [0,\infty) \to [0,\infty)$ is assumed for technical convenience to be continuously differentiable, strictly convex, with $\Gamma(0) = 0$, $\Gamma'(0) = 0$, and $\lim_{\rho\to\infty} \Gamma'(\rho) = \infty$.

The marketmaker's trading profit, per unit of time, is the product of the volume of trade, $\rho\mu_m$, and the bid-ask spread, $aX - bX$. Hence, a monopolistic marketmaker who searches with an intensity of ρ has an initial valuation of

$$\pi^M(\rho) = E\left[\int_0^\infty \rho\mu_m(t,\rho)\left(a(t,\rho) - b(t,\rho)\right) X_t e^{-rt}\, dt\right] - \frac{\Gamma(\rho)}{r}, \tag{16}$$

where $\mu_m = \min\{\mu_{ho}, \mu_{ln}\}$, and where we are using the obvious notation to indicate dependence of the solution on ρ and t.

Any one non-atomic marketmaker does not influence the equilibrium masses of investors, and therefore values his profit at

$$\pi^C(\rho) = \rho\, E\left[\int_0^\infty \mu_m(t)\left(a(t) - b(t)\right) X_t e^{-rt}\, dt\right] - \frac{\Gamma(\rho)}{r}.$$

An equilibrium intensity, ρ^C, for non-atomic marketmakers is a solution to the first-order condition

$$\Gamma'(\rho^C) = rE\left[\int_0^\infty \mu_m(t,\rho^C)\left(a(t,\rho^C) - b(t,\rho^C)\right) X_t e^{-rt}\, dt\right]. \tag{17}$$

The following theorem characterizes equilibrium search intensities in the case of "patient" marketmakers.

Theorem 8 *There exists a marketmaking intensity ρ^M that maximizes $\pi^M(\rho)$. There exists $\bar r > 0$ such that, for all $r < \bar r$ and for each $z \in [0,1]$, there exists a unique number $\rho^C(z)$ that solves (17), satisfying: $\rho^C(0) = 0$, $\rho^C(z)$ is increasing in z, and $\rho^C(1)$ is larger than any solution, ρ^M, to the monopolist's problem.*

In addition to providing the existence of equilibrium search intensities, this result establishes that: *(i)* competing marketmakers provide more marketmaking services if they can capture a higher proportion of the gains from trade, and *(ii)* competing marketmakers with full bargaining power provide more marketmaking services than a monopolistic marketmaker, since they do not internalize the consequences of their search on the masses of investor types. Notably, "small" marketmakers act strategically in their price setting (because all interactions are bilateral), but act "competitively" when choosing their levels of intermediation.

4.2 Welfare

We now consider the welfare implications of marketmaking in our search economy. We adopt a notion of "social welfare," the sum of the utilities of investors and marketmakers, which can be interpreted as the total investor utility in the case in which the marketmaker profits are redistributed to investors, for instance through share holdings. With our form of linear preferences, maximizing social welfare is a meaningful concept in that it is equivalent to requiring that utilities cannot be Pareto improved by changing allocations and by making initial consumption transfers.[12] By "investor welfare," we mean the total of investors' utilities, assuming that the marketmaker profits are not redistributed to investors. We take "marketmaker welfare" to be the total valuation of marketmaking profits, net of the cost of intermediation.

The welfare analysis is clearer if welfare losses are easily quantified. Hence, we assume that owners with adverse preference shocks, that is, agents of type ho, enjoy a dividend of $(1-\delta)X_t < X_t$, while other owners enjoy the full dividend, X_t, and that all agents have the same discount rate, $r = r_l = r_h$. The total "social-loss rate" is the cost rate $\Gamma(\rho)$ of intermediation plus the rate $\delta X_t \mu_{ho}(t)$ at which dividends are wasted through mis-allocation. At a given marketmaking intensity ρ, this leaves the social welfare

$$w^S(\rho) = E\left[\int_0^\infty \left(s - \delta\mu_{ho}(t)\right) X_t e^{-rt}\,dt\right] - \frac{\Gamma(\rho)}{r}.$$

Investor welfare is, similarly,

$$w^I(\rho) = E\left[\int_0^\infty \left(s - \delta\mu_{ho}(t,\rho) - \rho\mu_m(t,\rho)(a(t,\rho) - b(t,\rho))\right) X_t e^{-rt}\,dt\right],$$

[12]Also, this "utilitarian" social welfare function can be justified by considering the utility of an agent "behind the veil of ignorance," not knowing what type of agent he will become.

and the marketmakers' welfare is

$$w^M(\rho) = E\left[\int_0^\infty \rho\mu_m(t,\rho)(a(t,\rho) - b(t,\rho))X_t e^{-rt}\, dt\right] - \frac{\Gamma(\rho)}{r}.$$

We consider first the case of monopolistic marketmaking. We let ρ^M be the level of intermediation optimally chosen by the marketmaker, and ρ^S be the socially optimal level of intermediation. We note that the conditions imposed on Γ imply that there exists some $\bar{\rho} > 0$, independent of other parameters (with Γ fixed), such that $w^S(\rho) < 0$ and $w^M(\rho) < 0$ for all $\rho > \bar{\rho}$. The relation between the monopolistic marketmaker's chosen level ρ^M of intensity and the socially optimal intensity ρ^S is characterized in the following theorem.

Theorem 9 *(i) If investors cannot meet directly, that is, $\lambda = 0$, then the investor welfare $w^I(\rho)$ is independent of ρ, and a monopolistic marketmaker provides the socially optimal level ρ^S of intermediation (that is, $\rho^M = \rho^S$).*
(ii) If $\lambda > 0$, then $w^I(\rho)$ decreases in ρ when $\rho < \bar{\rho}$, and the monopolistic marketmaker over-invests in intermediation — that is, $\rho^M > \rho^S$, under any of the conditions: (a) *q is 0 or 1;* (b) *$r < \underline{r}$, for some $\underline{r} > 0$ depending on parameters other than ρ.*

The idea of this result is that, if investors cannot search, then their utilities do not depend on the level of intermediation because the monopolist extracts all gains from trade. In this case, because the monopolist gets all social benefits from providing intermediation and bears all the costs, he chooses the socially optimal level.

If, on the other hand, investors can trade directly with each other, then the marketmaker imposes a negative externality on investors, reducing their opportunities to trade directly with each other. Therefore, provided q is 0 or 1, investor welfare decreases with ρ. Consequently, the marketmaker's marginal benefit from intermediation is larger than the social benefit, so there is too much intermediation.

If $0 < q < 1$, then increasing ρ has the additional effect of changing the relative strength of investors' bargaining positions, because it changes their outside options. Some investors may benefit from this in the short run. In the long run (steady state), however, all investors are worse off with higher ρ. If agents have low discount rates then long-run effects dominate, and we get the result that the marketmaker over-invests in intermediation.

We now turn to the case of non-atomic (competing) marketmakers. In Section 4.1, we saw that the equilibrium level of intermediation of non-atomic marketmakers depends critically on their bargaining power. If they have no bargaining power, then they provide no intermediation. If they have all of the bargaining power, then they search more than a monopolistic marketmaker would.

A government may sometimes be able to affect intermediaries' market power, for instance through the enforcement of regulation (DeMarzo, Fishman, and Hagerty (2000)). Hence, we consider the following questions: How much marketmaker market power is socially optimal? How much market power would the intermediaries like to have? Would investors that marketmakers have market power? These questions are answered in the following proposition, in which we let z^I, z^S, and z^M denote the marketmaker bargaining power that would be chosen by, respectively, the investors, a social-welfare maximizing planner, and marketmakers.

Theorem 10 *It holds that $z^I > 0$. There is some $\underline{r} > 0$ such that, provided $r < \underline{r}$, we have $z^I < z^S \leq z^M = 1$.*

Investors in our model would prefer to enter a market in which non-atomic marketmakers have some market power, because this gives marketmakers an incentive to provide intermediation. The efficient level of intermediation is achieved with a higher market power to marketmakers. Marketmakers themselves prefer to have full bargaining power.

4.3 Explicit Bargaining Games

The setting considered here is the same as that of Section 2, with a few exceptions. First, agents can interact only at discrete moments in time, Δ_t apart. Later, we return to continuous time by letting Δ_t go to zero. Second, the bargaining game is modeled explicitly. Third, for simplicity we assume that there is no holding cost ($\delta = 0$).

We follow Rubinstein and Wolinsky (1985) and others in modeling an alternating-offers bargaining game, making the adjustments required by the specifics of our setup. When two agents are matched, one of them is chosen randomly, with probability $1/2$, to suggest a trading price. The other either rejects or accepts the offer, immediately. If the offer is rejected, the owner receives the dividend from the asset during the current period. At the next period, Δ_t later, one of the two agents is chosen at random, independently, to make a new offer. The bargaining may, however, break down before a counteroffer is made. A breakdown may occur because either of the agents changes discount rate, whence there are no longer gains from trade. A breakdown may also occur if one of the agents meets yet another agent, and leaves his current trading partner. The latter reason for breakdown is only relevant if agents are allowed to search while engaged in negotiation.

We consider first the case in which agents cannot search while bargaining. The offerer suggests the price that leaves the other agent indifferent between accepting and rejecting it. In the unique subgame perfect equilibrium, the offer is accepted immediately (Rubinstein (1982)). The value from rejecting is associated with the equilibrium strategies being played from then ownards. Letting $P_\sigma(X) = p_\sigma X$ be the price suggested by the agent of type σ with $\sigma \in \{ho, ln\}$, and letting $\bar{p} = (p_{ho} + p_{ln})/2$, we have

$$\begin{aligned}
p_{ln} + v_{hn} &= e^{-(r_h - c)\Delta_t}\big[\Delta_t + e^{-(\lambda_u+\lambda_d)\Delta_t}(\bar{p} + v_{hn}) \\
&\quad + e^{-\lambda_d\Delta_t}(1 - e^{-\lambda_u\Delta_t})v_{ho} + (1 - e^{-\lambda_d\Delta_t})v_{lo}\big] \\
-p_{ho} + v_{lo} &= e^{-(r_l - c)\Delta_t}\big[e^{-(\lambda_d+\lambda_u)\Delta_t}(-\bar{p} + v_{lo}) \\
&\quad + e^{-\lambda_u\Delta_t}(1 - e^{-\lambda_d\Delta_t})v_{ln} + (1 - e^{-\lambda_u\Delta_t})v_{hn}\big].
\end{aligned}$$

These prices, p_{ln} and p_{ho}, have the same limit $p = \lim_{\Delta_t \to 0} p_{ln} = \lim_{\Delta_t \to 0} p_{ho}$. Using (9), we obtain

$$p = \Delta v_h\,(1 - q) + \Delta v_l\,q, \tag{18}$$

where

$$q = \frac{r_l - c + \lambda_d + \lambda_u + 2\lambda\mu_{ho}}{r_l + r_h - 2c + 2(\lambda_d + \lambda_u) + 2\lambda\mu_{ho} + 2\lambda\mu_{ln}}. \tag{19}$$

This formula (19) for the endogenous bargaining power highlights the fact that an agent's ability to meet alternative trading partners makes him more impatient, decreasing his bargaining power. A high ability to meet alternative trading partners increases the outside option, however, which gives an indirect advantage.

Suppose, instead, that agents can search for alternative trading partners during negotiations, and that, given contact with an alternative partner, they leave the present partner in order to negotiate with the newly found one. This model is solved similarly to the previous one. In the limit, as

$\Delta_t \to 0$, the price is given by (18), where

$$q = \frac{r_l - c + \lambda_d + \lambda_u + 2\lambda\mu_{ho} + 2\lambda\mu_{ln}}{r_l + r_h - 2c + 2(\lambda_d + \lambda_u + 2\lambda\mu_{ho} + 2\lambda\mu_{ln})}\,. \tag{20}$$

Here, one agent's intensity of meeting other trading partners influences the bargaining power of both agents in the same way. This is because one's own ability to meet an alternative trading partner: *(i)* makes oneself more impatient, and *(ii)* also increases the partner's risk of breakdown.

One can model explicitly the interaction between marketmakers and investors in a similar alternating-offers game. For this, one must define the marketmakers' discount rate. We do not document the results here, since they are quite messy and do not shed much additional light, but we remark that the solution is of the form stipulated in Section 3.

In this section, we have found a subgame-perfect bargaining equilibrium and derived explicit formulae for the bargaining power, q, showing that the transaction price depends on agents' outside options in the linear way that we specify. (See Footnote 8 for further discussion.) Qualitatively, most of our results with exogenous bargaining power are unchanged if the bargaining power is endogenized as in (20), and we will not extend them here. It is interesting to note, however, that if we use (19) to endogenize the bargaining power, then, for instance, q approaches 0 or 1 as λ increases. Furthermore, q tends to 0 precisely when convergence to the Walrasian equilibrium requires it to be bounded below away from 0, that is, under Condition 1. The limiting price as λ tends to infinity is not Walrasian in this case. (An analogous property holds for q approaching 1.)

4.4 Asymmetric Information

It is natural that information about future dividends held privately by agents may be transmitted through trading. If agents observe only their own transactions, one would expect that the speed with which information is spread is related to agents' search intensities. According to this intuition, information is always disseminated, although slowly, if search intensities are low. We show, however, that this need *not* be the case. If meeting intensities are low, agents are eager to trade when they meet since they know that failure to trade is particularly costly. This leads to the existence of pooling equilibria in which *no* information is revealed through trading. We show that such pooling equilbria exist only for sufficiently small search intensities. We do not study equilibria in which information is disseminated through bargaining interaction, as did Wolinsky (1990), although this would also be interesting.

We model asymmetric information as follows. The dividend process X jumps with a known constant jump-arrival intensity λ_J, so that at any jump time τ, the relative jump size $X(\tau)(X(\tau-))^{-1}$ is drawn independently of $X(\tau-)$ and of agents' types. The relative jump size is drawn with probability $1-\gamma$ from a distribution with mean J_0, and with probability γ from a distribution with mean $J_1 > J_0$. The unconditional mean relative jump size, consequently, is $J_m = \gamma J_1 + (1-\gamma)J_0$. Suppose further that, in the event that the next relative jump is to be drawn with the high conditional mean, a proportion $\nu \in [0,1]$ of the agents, independently selected, are informed of this fact immediately after the previous jump. The allocation of this information is independent of X, and of agents' current types. In the event that the relative jump is to be drawn with the low conditional mean, nobody receives information regarding this fact. Thus, each agent is informed with probability $\gamma\nu$, and an uninformed agent expects a relative jump of conditional mean

$$J^u = \frac{\gamma(1-\nu)J_1 + (1-\gamma)J_0}{1-\gamma\nu}.$$

In order to keep our analysis relatively simple, we assume that, once two agents meet, one of them is drawn randomly to make a take-it-or-leave-it offer. We use the notation q_σ for the probability that an agent of type σ is the quoting agent. We are looking for conditions under which there is a pooling equilibrium, in which sellers quote a price at which both informed and uninformed buyers are willing to buy, rather than a more aggressive price at which uninformed buyers would decline trade. Likewise, buyers quote pooling prices. Before we determine these pooling prices, we point out that our pooling equilibrium also requires that agents with no gains from trade must not reveal information by trading with each other. This is, however, consistent with optimal behavior. For instance, an uninformed owner with a low discount rate does not sell to an informed agent with low discount rate, since there are no gains from trade between the two. If such a trade took place, then the uninformed would become informed, but the expected utility of these agents would remain unchanged.[13] Such trades are ruled out, for instance, if there is an arbitrarily small cost of making an offer.

[13]We note, however, that in a partially revealing equilibrium, in which being informed would be valuable for future behavior, there would exist strictly positive gains from such a trade.

We now turn to the determination of the value functions and pooling prices. We refine the notation of Section 2.2 by appending to the value coefficient v_σ a superscript "i" if the agent is informed, and a superscript "u" otherwise. We also define the reservation-value coefficients for each of the four cases as follows: $\Delta v_h^i = v_{ho}^i - v_{hn}^i$, $\Delta v_h^u = v_{ho}^u - v_{hn}^u$, $\Delta v_l^i = v_{lo}^i - v_{ln}^i$, and $\Delta v_h^i = v_{lo}^u - v_{ln}^u$. We look for equilibria in which, naturally, informed agents have higher reservation values than those of uninformed agents, and all efficient trade can potentially happen, that is,

$$\Delta v_l^i \geq \Delta v_l^u \geq \Delta v_h^i \geq \Delta v_h^u. \tag{21}$$

Proposition 12 in Appendix A offers mild sufficient conditions for (21). A full equilibrium analysis, including the system of linear equations analogous to those of Section 2.2, is found in Appendix A.

Here, we present only the necessary and sufficient conditions for a pooling equilibrium. First, a high-discount-rate owner, whether informed or not, must prefer to quote a price which is accepted by all non-owners with a low discount rate, rather than quoting a more aggressive price, which would be accepted only by informed non-owners. That is,

$$\begin{aligned} \Delta v_l^u + v_{hn}^i &\geq Pr(i\,|\,i)\left(\Delta v_l^i + v_{hn}^i\right) + (1 - Pr(i\,|\,i))v_{ho}^i\,, \qquad (22)\\ \Delta v_l^u + v_{hn}^u &\geq Pr(i\,|\,u)\left(\Delta v_l^i + v_{hn}^u\right) + (1 - Pr(i\,|\,u))v_{ho}^u, \qquad (23)\end{aligned}$$

where $Pr(i\,|\,\xi)$ is the probability of the buyer being informed given that the seller has information status $\xi \in \{i, u\}$. The left-hand side of (22) is the value to an informed high-discount-rate owner of quoting the pooling price, Δv_l^u (given that there are gains from trade with this counterparty). The right-hand side is the value of quoting the most aggressive price, Δv_l^i, namely the reservation value of an informed non-owner (again, given that there are gains from trade with this counterparty). Similarly, (23) states that an uninformed high-discount-rate owner prefers to quote the pooling price. We note that (22)–(23) must be satisfied for any pooling equilibrium, regardless of the out-of-equilibrium beliefs. One possible choice of out-of-equilibrium beliefs is that conditional on any out-of-equilibrium price offer, the expected jump mean of an uninformed remains J^u.

Also, a low-discount-rate non-owner, whether informed or not, must prefer to buy at the pooling price with certainty rather than buying at a lower price only from uninformed sellers, that is,

$$\begin{aligned} v_{lo}^i - \Delta v_h^i &\geq Pr(u\,|\,i)\left(v_{lo}^i - \Delta v_h^u\right) + (1 - Pr(u\,|\,i))\,v_{ln}^i \qquad (24)\\ v_{lo}^u - \Delta v_h^i &\geq Pr(u\,|\,u)\,(v_{lo}^u - \Delta v_h^u) + (1 - Pr(u\,|\,u))\,v_{ln}^u. \qquad (25)\end{aligned}$$

It turns out that only the optimality conditions of the informed seller (22), and of the uninformed buyer (25) need to be checked. If these two conditions are satisfied, the other two optimality conditions follow automatically. (Proposition 12 in Appendix A formalizes this claim.)

For a given set of parameters, either of the necessary and sufficient optimality conditions, (22) and (25), may or may not hold. Intuitively, the first condition fails when, keeping all other parameters fixed, there are "so many" informed agents (ν is sufficiently high) that an (informed) seller would benefit by quoting an aggressive price and risking the loss of a trade with an uninformed agent. Similarly, the second condition fails when, keeping all other parameters fixed, an (uninformed) buyer perceives the proportion of uninformed agents as too large (ν is sufficiently small). When search is too intense, there is no pooling equilibrium, and information must be revealed through trading:

Theorem 11 *For any set of parameters, there exists a search intensity $\bar{\lambda}$ such that, for all $\lambda > \bar{\lambda}$, a pooling equilibrium cannot exist.*

When search is less intense, however, pooling equilibria may exist for an open set of parameters. Figure 7 provides an illustrative numerical example. We use the parameters of Table 1 and take $J_0 = 1$, $J_1 = 1.1$, $\lambda_J = 0.2$, and $\gamma = 0.8$. We compute, for a range of contact intensities (λ), the minimal and maximal proportion of informed agents, ν, consistent with a pooling equilibrium. We see that, as λ increases, ν is confined to a smaller and smaller interval, depicted as the shaded region of Figure 7, until the two optimality conditions (22) and (25) can no longer be satisfied simultaneously. One can see that the seller's incentive constraint for pooling is more sensitive to λ than the buyer's, because the buy side of the market is larger than the sell side (Condition 1 is satisfied). Hence, as λ increases, a seller's meeting intensity converges to infinity, which makes it tempting for the seller to quote aggressive prices. The buyer's meeting intensity, on the other hand, is bounded as λ increases.

A Appendix: Proofs

Proof of Propositions 1 and 4:

First note that Proposition 1 is a special case of Proposition 4 with $\rho = 0$.

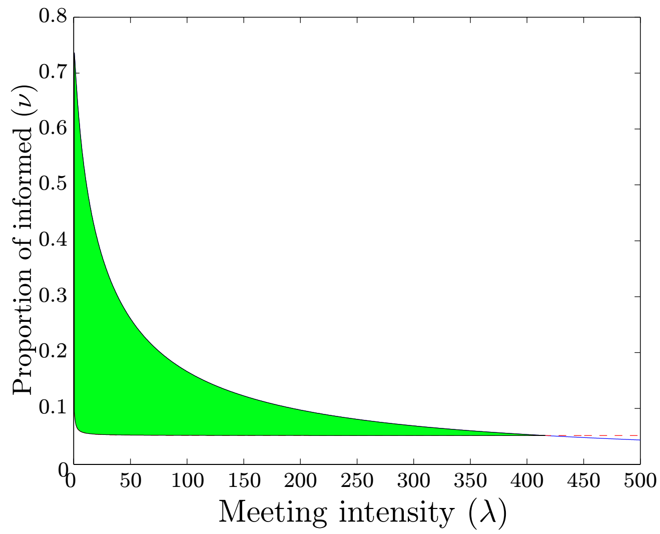

Figure 7: The shaded area is the set of parameters for which a pooling equilibrium exists. The solid line shows the highest value that ν can take, while preserving pooling condition (22) for quotation by informed sellers. The dotted line shows the lowest value of ν consistent with the pooling condition (25) of uninformed buyers.

Let

$$y = \frac{\lambda_d}{\lambda_d + \lambda_u},$$

and assume that $y > s$. (This is Condition 1.) The case $y \leq s$ can be treated analogously. Setting the right-hand side of Equation 3 to zero and substituting all components of μ other than μ_{ho} in terms of μ_{ho} from Equations (1) and (2) and from $\mu_{ho} + \mu_{hn} = \lambda_u(\lambda_u + \lambda_d)^{-1} = 1 - y$, we obtain the quadratic equation

$$Q(\mu_{ho}) = 0,$$

where

$$Q(x) = 2\lambda x^2 + (2\lambda(y - s) + \rho + \lambda_d + \lambda_u)x - \lambda_u s. \tag{A.1}$$

It is immediate that Q has a negative root (since $Q(0) < 0$) and has a root in the interval $(0, 1)$ (since $Q(1) > 0$).

Since μ_{ho} is the largest and positive root of a quadratic with positive leading coefficient and with a negative root, in order to show that $\mu_{ho} < \eta$ for some $\eta > 0$ it suffices to show that $Q(\eta) > 0$. Thus, in order that $\mu_{lo} > 0$

(for, clearly, $\mu_{lo} < 1$), it is sufficient that $Q(s) > 0$, which is true, since

$$Q(s) = 2\lambda s^2 + (\lambda_d + \rho)s.$$

Similarly, $\mu_{hn} > 0$ if $Q(1-y) > 0$, which holds because

$$Q(1-y) = 2\lambda(1-y)^2 + 2\lambda(y-s) + (\lambda_u + \rho)(1-s).$$

Finally, since $\mu_{ln} = y - s + \mu_{ho}$, it is immediate that $\mu_{ln} > 0$.

We present a sketch of a proof of the claim that, from any admissible initial condition $\mu(0)$ the system converges to the steady-state μ.

Because of the two restrictions (1) and (2), the system is reduced to two equations, which can be thought of as equations in the unknowns $\mu_{ho}(t)$ and $\mu_h(t)$, where $\mu_h(t) = \mu_{ho}(t) + \mu_{hn}(t)$. The equation for $\mu_h(t)$ does not depend on $\mu_{ho}(t)$, and admits the simple solution:

$$\mu_h(t) = \mu_h(0)e^{-(\lambda_u+\lambda_d)t} + \frac{\lambda_u}{(\lambda_u+\lambda_d)}(1 - e^{-(\lambda_u+\lambda_d)t}).$$

Define the function

$$G(w,x) = -2\lambda x^2 - (\lambda_d + \lambda_u + 2\lambda(1-s-w) + \rho)x + \rho \max\{0, s+w-1\} + \lambda_u s$$

and note that μ_{ho} satisfies

$$\dot{\mu}_{ho}(t) = G(\mu_h(t), \mu_{ho}(t)).$$

The claim is proved by the steps:

1. Choose t_1 high enough that $1 - s - \mu_h(t)$ does not change sign for $t > t_1$.

2. Show that $\mu_{ho}(t)$ stays in $(0,1)$ for all t, by verifying that $G(w,0) > 0$ and $G(w,1) < 0$.

3. Choose t_2 ($\geq t_1$) high enough that $\mu_h(t)$ changes by at most an arbitrarily chosen $\epsilon > 0$ for $t > t_2$.

4. Note that, for any value $\mu_{ho}(t_2) \in (0,1)$, the equation

$$\dot{x}(t) = G(w, x(t)) \tag{A.2}$$

admits a solution that converges exponentially, as $t \to \infty$, to a positive quantity that can be written as $(-b + \sqrt{b^2 + 4ac})/2a$, where only b and c depend on w, and in an affine fashion. The convergence is uniform in $\mu_{ho}(t_2)$.

5. Finally, using a comparison theorem (for instance, see Birkhoff and Rota (1969), page 25), $\mu_{ho}(t)$ is bounded by the solutions to (A.2) corresponding to w taking the highest and lowest values of $\mu_h(t)$ for $t > t_2$ (these are, of course, $\mu_h(t_2)$ and $\lim_{t\to\infty} \mu_h(t)$). By virtue of the previous step, for high enough t, these solutions are within $O(\epsilon)$ of the steady-state solution μ_{ho}.

□

Proof of Theorem 2:

We present here a sketch of the proof. The issue is to show that any agent prefers, at any time, given all information, to play the proposed equilibrium trading strategy, assuming that other agents do. It is enough to show that an agent agrees to trade at the candidate equilibrium prices when contacted by an investor with whom there are potential gains from trade. Our calculations in Section 2, and the assumption that $c < r_l$, already imply that the value function is equal to the utility of the consumption process generated by the candidate trading strategy, at the candidate prices. We must now check that any other trading strategy generates no higher utility.

The Bellman principle, when applied at a time when the dividend rate is x, for an agent of type ho in contact with an agent of type ln, is that: Selling the asset, consuming the price, and attaining the candidate value of a non-owner with a high discount rate, dominates (at least weakly) the value of keeping the asset, consuming its dividends and collecting the discounted expected candidate value achieved at the next time τ_m of a trading opportunity or at the next time τ_r of a change to a low discount rate, whichever comes first. That is, for an agent of type ho,

$$P(x) + V(x, hn) \geq E\bigg[\int_0^\tau xe^{ct}e^{-r_h t}\, dt$$

$$+ e^{-r_h\tau}\left[(V(xe^{c\tau}, hn) + P(xe^{c\tau}))\, 1_{\{\tau=\tau_m\}} + V(xe^{c\tau}, lo)1_{\{\tau=\tau_r\}}\right]\bigg],$$

where $\tau = \min(\tau_r, \tau_m)$. There is a like Bellman inequality for an agent of type ln. Both of these inequalities are satisfied in our candidate equilibrium.

Now, to verify the sufficiency of the Bellman equations for individual optimality, consider any initial agent type σ_0, any feasible trading strategy, N, an adapted process whose value is 1 whenever the agent owns the asset and

0 whenever the agent does not own the asset. The cumulative consumption process C^N associated with this trading strategy is given by

$$dC_t^N = N_t X_t \left(1 - \delta 1_{\{\sigma(t)=ho\}}\right) \, dt - pX_t \, dN_t. \tag{A.3}$$

The type process associated with trading strategy N is denoted σ^N.

Following the usual verification argument for stochastic-control, for any future time T,

$$V(x, \sigma_0) \geq E\left[\int_0^T e^{-\int_0^t R(\sigma_s^N)\, ds} \, dC_t^N\right] + E\left[e^{-\int_0^T R(\sigma_s^N)\, ds} \, V(X_T, \sigma_T^N)\right],$$

where $R(hn) = R(ho) = r_h$ and $R(ln) = R(lo) = r_l$. (This assumes without loss of generality that a potential trading contact does not occur at time 0.) Letting T go to ∞ and using $c < r_l$, we have $V(x, \sigma_0) \geq U(C^N)$. Because $V(x, \sigma) = U(C^*)$, where C^* is the consumption process associated with the candidate equilibrium strategy, we have shown optimality.

□

Analysis of agents' reservation values:

A simple modification of (7) allows for the treatment of the case with non-atomic marketmakers, who have an arbitrary bargaining power, $z \in [0, 1]$. Note that, as described in Section 3.1, special cases are the case of no marketmakers, $\rho = 0$, and the case of a monopolistic marketmaker, $z = 1$. Here, we derive some general results that are used in the proofs below.

Note that, under Condition 1, only a proportion, μ_{ho}/μ_{ln}, of the agents of type ln buy from the marketmaker, when they meet him. Let $\rho' = \rho \mu_{ho} \mu_{ln}^{-1}$. The equations for the coefficients of the value functions and prices are:

$$\begin{aligned}
v_{ho} &= \frac{(\lambda_d v_{lo} + 2\lambda \mu_{ln} p + \rho b + (2\lambda \mu_{ln} + \rho) v_{hn} + 1 - \delta)}{r_h + \lambda_d + 2\lambda \mu_{ln} + \rho - c} \\
v_{hn} &= \frac{\lambda_d v_{ln}}{r_h + \lambda_d - c} \\
v_{lo} &= \frac{(\lambda_u v_{ho} + 1)}{r_l + \lambda_u - c} \\
v_{ln} &= \frac{(\lambda_u v_{hn} + (2\lambda \mu_{ho} + \rho') v_{lo} - 2\lambda \mu_{ho} p - \rho' a)}{r_l + \lambda_u + 2\lambda \mu_{ho} + \rho' - c} \\
p &= (v_{ho} - v_{hn})(1 - q) + (v_{lo} - v_{ln}) q \\
a &= v_{lo} - v_{ln} \\
b &= (v_{ho} - v_{hn}) z + (v_{lo} - v_{ln})(1 - z).
\end{aligned}$$

Define $\Delta v_h = v_{ho} - v_{hn}$ and $\Delta v_l = v_{lo} - v_{ln}$ to be the reservation-value coefficients. The bargaining power of a seller who interacts with a marketmaker is $1 - z$, while buyers pay their reservation values. Appropriate linear combinations of the equations above yield

$$W_1 \psi \;=\; (1 - \delta, 1)^\top, \tag{A.4}$$

where $\psi = (\Delta v_h, \Delta v_l)^\top$, and

$$W_1 \;=\; \begin{bmatrix} r_h - c + \lambda_d + 2\lambda\mu_{ln} q + \rho(1-z) & -(\lambda_d + 2\lambda\mu_{ln} q + \rho(1-z)) \\ -(\lambda_u + 2\lambda\mu_{ho}(1-q)) & r_l - c + \lambda_u + 2\lambda\mu_{ho}(1-q) \end{bmatrix}.$$

It will be used repeatedly in what follows that

$$\Delta v_l - \Delta v_h = \frac{r_h - r_l + \delta(r_l - c)}{\det(W_1)} > 0. \tag{A.5}$$

Proof of Theorems 3 and 6:

In the context of Theorem 3, $\rho = 0$. In the context of Theorem 6, $z = 1$, which implies that the term $\rho(1-z)$ is 0 (even when "$\rho = \infty$", i.e., when marketmakers are instantaneously accessible). Equation (A.5) shows that $\Delta v_l - \Delta v_h \to 0$ if and only if $\det(W_1) \to \infty$. The latter happens if and only if $\lambda\mu_{ln} q \to \infty$, since, under Condition 1, $\lambda\mu_{ho}(1-q)$ is bounded. That $\lambda\mu_{ln} q \to \infty$ follows from $q > 0$ and $\mu_{ln} \geq y - s > 0$ (again, under Condition 1). Using the second row of equation (A.4), one deduces that $\Delta v_h \to (r_l - c)^{-1}$ and that $\Delta v_l \to (r_l - c)^{-1}$.

It is clear from (A.1) that $\mu_{ho}^k \to 0$, which implies that $\mu^k \to \mu^*$.

□

Proof of Theorem 5:

It is immediate from (A.1) that, as $\rho \to \infty$, $\mu_{ho} \to 0$. The limit of ψ^k is obtained from (A.4) with $z = 1$, $\mu_{ln} = y - s$, and $\mu_{ho} = 0$. These same equations, (A.1) and (A.4), characterize the prices set by a monopolistic marketmaker that can be approached instantly. Therefore, the reservation-value coefficients, and hence the bid and ask coefficients, converge to the monopolistic bid and ask coefficients.

In order to show that $a - b$ increases in ρ, it suffices to prove that the determinant of W_1 decreases in ρ, which is true because the masses μ_{ln} and μ_{ho} do.

□

Proof of Theorem 7:

Since $1 - z > 0$, the determinant of W_1 tends to infinity as ρ increases, whence $\Delta v_l^k - \Delta v_h^k \to 0$ (by (A.5)). As in the proof of Theorem 3, the common limit of the two sequences is the Walrasian price coefficient p^*.

□

Proof of Theorem 8:

There exists a number, ρ^M, that maximizes (16) since $\pi^M(\,\cdot\,)$ is continuous and $\pi^M(\rho) \to -\infty$ as $\rho \to \infty$.

We are looking for a $\rho^C \geq 0$ such that

$$\Gamma'(\rho^C) = rE \int_0^\infty \mu_m(\rho^C)(a(\rho^C) - b(\rho^C))X_t e^{-rt}\, dt. \tag{A.6}$$

Consider how both the left- and right-hand sides depend on ρ. The left-hand side is 0 for $\rho = 0$, increasing, and tends to infinity as ρ tends to infinity. The right-hand side (RHS) is strictly positive for $\rho = 0$. Further, the steady-state value of the RHS can be seen to be decreasing, using that μ_m is decreasing in ρ, and using the explicit expression for the spread given by (A.5). The closer the discount rate r is to c, the more important the steady-state value becomes to the determination of the sign of the integral. Therefore, the RHS is also decreasing in ρ for any initial condition of μ if r is small enough. These results yield the existence of a unique solution.

For $z = 0$, $b(t, \rho) - a(t, \rho) = 0$ everywhere, so the solution to (A.6) is $\rho^C = 0$. To see that $\rho^C > \rho^M$ when $z = 1$, consider the first-order conditions that determine ρ^M:

$$\begin{aligned} \Gamma'(\rho^M) \;\; = \;\; & rE \int_0^\infty \Big[\mu_m(t, \rho^M)(a(t, \rho^M) - b(t, \rho^M)) \\ & + \rho^M \frac{\partial}{\partial \rho^M}\left(\mu_m(t, \rho^M)(a(t, \rho^M) - b(t, \rho^M))\right)\Big] X_t e^{-rt}\, dt. \end{aligned} \tag{A.7}$$

The integral of the first integrand term on the right-hand side of (A.7) is the same as that of (A.6), and that of the second is negative for small r. Hence, the right-hand side of (A.7) is smaller than the right-hand side of (A.6), implying that $\rho^C(1) > \rho^M$.

To see that $\rho^C(z)$ is increasing in z, we use the Implicit Function Theorem and the dominated convergence theorem to compute the derivative of $\rho^C(z)$ with respect to z, as

$$\frac{rE\int_0^\infty \mu_m(\rho^C)(a_z(\rho^C,z)-b_z(\rho^C,z))X_t e^{-rt}\,dt}{\Gamma''(\rho^C)-rE\int_0^\infty \frac{d}{d\rho}\mu_m(\rho^C)(a(\rho^C,z)-b(\rho^C,z))X_t e^{-rt}\,dt}. \tag{A.8}$$

If we use the steady-state expressions for μ, a, and b, this expression is seen to be positive because both the denominator and the numerator are positive. Hence, it is also positive with any initial masses if we choose r small enough.

□

Proof of Theorem 9: *(i)* The first part of the theorem, that the monopolistic marketmaker's search intensity does not affect investors when they can't search for each other, is obvious. Indeed, each investor's utility is that derived in autarky.

(ii) Letting $\Delta v_o = v_{lo} - v_{ho}$, $\Delta v_n = v_{ln} - v_{hn}$, and $\phi = \Delta v_o - \Delta v_n$, we start by proving a few general facts about the marketmaker spread, ϕ.

The dynamics of ϕ are given by the ordinary differential equation

$$\dot{\phi}_t = (r+\lambda_u+\lambda_d+2\lambda(1-q)\mu_{ho}+2\lambda q\mu_{ln})\phi_t - \delta,$$

Let $R = r+\lambda_u+\lambda_d+2\lambda(1-q)\mu_{ho}+2\lambda q\mu_{ln}$. The equation above readily implies that

$$\frac{\partial\dot{\phi}_t}{\partial\rho} = R\frac{\partial\phi_t}{\partial\rho} + \left(2\lambda(1-q)\frac{\partial\mu_{ho}(t)}{\partial\rho} + 2\lambda q\frac{\partial\mu_{ln}(t)}{\partial\rho}\right)\phi_t. \tag{A.9}$$

A simple comparison argument immediately yields that $\frac{\partial\mu_{ho}(t)}{\partial\rho} = \frac{\partial\mu_{ln}(t)}{\partial\rho} < 0$, whence $\frac{\partial\phi}{\partial\rho} > 0$.

(a) Consider now the case $q = 1$, for which

$$\dot{v}_{lo}(t) = rv_{lo}(t) + \lambda_d\phi_t,$$

which, since $\frac{\partial\phi_t}{\partial\rho} > 0$, implies that $\frac{\partial v_{lo}(t)}{\partial\rho} < 0$. Consequently, $v_{ho}(t) = v_{lo}(t) - \phi_t$ also decreases in ρ.

If $q = 0$,

$$\dot{v}_{hn}(t) = rv_{hn}(t) + \lambda_d(\phi_t - \Delta v_o(t)),$$

and since $\Delta v_o(t)$ is independent of ρ, $v_{hn}(t)$ decreases in ρ. Consequently, $v_{ln}(t) = v_{hn}(t) - \phi_t + \Delta v_o(t)$ also decreases in ρ.

(b) The claim follows from the fact that the derivative with respect to ρ of the steady-state flow to investors, $\delta\mu_{ho} + \mu_{ho}\rho\phi$, is negative. Using continuity in (t, ρ), one concludes that the derivative of the flow at time t is bounded above away from zero for all $\rho < \bar{\rho}$ and $t > T$ for some T. Consequently, some $\bar{r}$ exists with the property stated by the theorem.

□

Proof of Theorem 10:

To see that $z^I > 0$, we note that with $\rho = \rho^C(z)$,

$$\frac{d}{dz} w^I \mid_{z=0} \;=\; -\delta E \int_0^\infty \frac{d}{d\rho}\mu_{ho}(t, \rho) X_t e^{-rt}\, dt\, \frac{d\rho^C}{dz} > 0,$$

where we have used that $\rho^C(0) = 0$, that $\frac{d\rho^C}{dz} > 0$ at $z = 0$ (see (A.8)), that $a - b = 0$ if $z = 0$, and that for all t, $\frac{d}{d\rho}\mu_{ho}(t, \rho) < 0$.

To prove that $z^I < z^S \leq z^M = 1$, it suffices to show that the marketmaker welfare is increasing in z, which follows from

$$\begin{aligned}
\frac{d}{dz} w^M &= \rho \frac{d}{dz}\left[E \int_0^\infty \mu_{ho}(a - b) X_t e^{-rt}\, dt\right] \\
&= \frac{\rho}{r} \frac{d}{dz} \Gamma'(\rho^C(z)) \\
&= \frac{\rho}{r} \Gamma''(\rho^C(z)) \frac{d\rho^C}{dz} > 0,
\end{aligned}$$

suppressing the arguments t and ρ from the notation, where we have used twice that $\Gamma'(\rho) = rE \int_0^\infty \mu_{ho}(a - b) X_t e^{-rt}\, dt$ if $\rho = \rho^C(z)$, and that $\frac{d\rho^C}{dz} > 0$ (Theorem 8).

□

Analysis of pooling equilibria with information: We work under condition (21) in the text, which means that prices are set by the reservation values of the informed seller and uninformed buyer, and that the bid is higher than the ask. Under these conditions, one derives the equations:

$$
\begin{aligned}
v^i_{ho} &= \left(\lambda_d v^i_{lo} + 2\lambda\mu_{ln}(p + v^i_{hn}) + \lambda_J J_1(\gamma\nu v^i_{ho} + (1-\gamma\nu)v^u_{ho}) + 1 - \delta\right) \cdot \\
&\quad \frac{1}{r_h + \lambda_d + 2\lambda\mu_{ln} + \lambda_J - c} \\
v^i_{hn} &= (\lambda_d v^i_{ln} + \lambda_J J_1(\gamma\nu v^i_{hn} + (1-\gamma\nu)v^u_{hn}))\frac{1}{r_h + \lambda_d + \lambda_J - c} \\
v^i_{lo} &= \left(\lambda_u v^i_{ho} + \lambda_J J_1(\gamma\nu v^i_{lo} + (1-\gamma\nu)v^u_{lo}) + 1\right)\frac{1}{r_l + \lambda_u + \lambda_J - c} \\
v^i_{ln} &= \left(\lambda_u v^i_{hn} + 2\lambda\mu_{ho}(v^i_{lo} - p) + \lambda_J J_1(\gamma\nu v^i_{ln} + (1-\gamma\nu)v^u_{ln})\right) \cdot \\
&\quad \frac{1}{r_l + \lambda_u + 2\lambda\mu_{ho} + \lambda_J - c} \\
v^u_{ho} &= \left(\lambda_d v^u_{lo} + 2\lambda\mu_{ln}(p + v^i_{hn}) + \lambda_J J^u(\gamma\nu v^i_{ho} + (1-\gamma\nu)v^u_{ho}) + 1 - \delta\right) \cdot \\
&\quad \frac{1}{r_h + \lambda_d + 2\lambda\mu_{ln} + \lambda_J - c} \qquad\qquad (A.10)\\
v^u_{hn} &= (\lambda_d v^u_{ln} + \lambda_J J^u(\gamma\nu v^i_{hn} + (1-\gamma\nu)v^u_{hn}))\frac{1}{r_h + \lambda_d + \lambda_J - c} \\
v^u_{lo} &= \left(\lambda_u v^u_{ho} + \lambda_J J^u(\gamma\nu v^i_{lo} + (1-\gamma\nu)v^u_{lo}) + 1\right)\frac{1}{r_l + \lambda_u + \lambda_J - c} \\
v^u_{ln} &= \left(\lambda_u v^u_{hn} + 2\lambda\mu_{ho}(v^u_{lo} - p) + \lambda_J J^u(\gamma\nu v^i_{ln} + (1-\gamma\nu)v^u_{ln})\right) \cdot \\
&\quad \frac{1}{r_l + \lambda_u + 2\lambda\mu_{ho} + \lambda_J - c} \\
p &= (v^i_{ho} - v^i_{hn})(1-q) + (v^u_{lo} - v^u_{ln})q.
\end{aligned}
$$

Here, p represents the expected price coefficient; the realized price coefficient is $v^i_{ho} - v^i_{hn}$ or $v^u_{lo} - v^u_{ln}$.

Proposition 12 *If $J_1 - J_0 < 1/\gamma\nu$, the solution to the linear system (A.10) satisfies $\Delta v^i_l \geq \Delta v^u_l$ and $\Delta v^i_h \geq \Delta v^u_h$. If the solution to the linear system (A.10) satisfies $\Delta v^i_l \geq \Delta v^u_l \geq \Delta v^i_h \geq \Delta v^u_h$, then conditions (22) and (25) ensure that this solution defines a pooling equilibrium.*

Proof: Let us first prove the first part of the proposition, namely that the solution to the system above satisfies $v^i_{ho} - v^i_{hn} \geq v^u_{ho} - v^u_{hn}$ and $v^i_{lo} - v^i_{ln} \geq v^u_{lo} - v^u_{ln}$. To that end, recall the definitions $\Delta v^i_h = v^i_{ho} - v^i_{hn}$, $\Delta v^u_h = v^u_{ho} - v^u_{hn}$, $\Delta v^i_l = v^i_{lo} - v^i_{ln}$, and $\Delta v^u_l = v^u_{lo} - v^u_{ln}$. Let $\phi_h = \Delta v^i_h - \Delta v^u_h$ and $\phi_l = \Delta v^i_l - \Delta v^u_l$. By adding and subtracting appropriately the equations above, one obtains

$$
\begin{aligned}
\phi_h(r_h - c + \lambda_d + 2\lambda\mu_{ln} + \lambda_J) &= \phi_h\gamma\nu\lambda_J(J_1 - J^u) + \phi_l\lambda_d + \lambda_J(J_1 - J^u)\Delta v^u_h \\
\phi_l(r_l - c + \lambda_u + 2\lambda\mu_{ho} + \lambda_J) &= \phi_l\gamma\nu\lambda_J(J_1 - J^u) + \phi_h\lambda_u + \lambda_J(J_1 - J^u)\Delta v^u_l.
\end{aligned}
$$

This system of equations is guaranteed to have a positive solution in (ϕ_h, ϕ_l) when the operator norm of the matrix

$$W_2 = \begin{bmatrix} \frac{\gamma\nu\lambda_J(J_1-J^u)}{r_h-c+\lambda_d+2\lambda\mu_{ln}+\lambda_J} & \frac{\lambda_d}{r_h-c+\lambda_d+2\lambda\mu_{ln}+\lambda_J} \\ \\ \frac{\lambda_u}{r_l-c+\lambda_u+2\lambda\mu_{ho}+\lambda_J} & \frac{\gamma\nu\lambda_J(J_1-J^u)}{r_l-c+\lambda_u+2\lambda\mu_{ho}+\lambda_J} \end{bmatrix}$$

is strictly less than 1. The proof also relies on the positivity of all the coefficients of the system, which makes Brouwer's Theorem applicable. Since all entries of W_2 are positive, it suffices that the sums of the elements of each row be smaller than 1 in order to get $\|W_2\| < 1$. This condition follows when J_1 is not much larger than J_0; for instance, $J_1 - J_0 < 1/\gamma\nu$ is sufficient for our purposes.

Let us now turn to the second claim of the proposition. Consider a seller with information status $\theta \in \{i, u\}$. The seller's bargaining power does not matter, since we assume that it is captured by an independent random draw that determines which side makes the "take-it-or-leave-it" offer. This analysis conditions on the event that the seller makes the offer. Equations (22) and (23) can be written as

$$\Delta v_l^u \geq \Delta v_l^i Pr(i \,|\, \theta) + \Delta v_h^\theta \left(1 - Pr(i \,|\, \theta)\right).$$

In order to show that the constraint for $\theta = i$ is stronger than the constraint for $\theta = u$, it suffices to show that

$$\Delta v_l^i Pr(i \,|\, i) + \Delta v_h^i Pr(u \,|\, i) \geq \Delta v_l^i Pr(i \,|\, u) + \Delta v_h^u Pr(u \,|\, u),$$

which is equivalent to

$$(\Delta v_l^i - \Delta v_h^i) Pr(u \,|\, i) \leq (\Delta v_l^i - \Delta v_h^u) Pr(u \,|\, u),$$

which in turn holds because $\Delta v_h^i \geq \Delta v_h^u$ and $Pr(u \,|\, i) \leq Pr(u \,|\, u)$.

Analogously, one deduces that the uninformed-buyer condition is stronger than the informed-buyer condition. Consequently, if (22) and (25) hold, then (23) and (24) also do, whence quoting pooling prices is optimal for all agents, given that everybody else does the same. This proves that the solution to (A.10) defines a pooling equilibrium.

□

Proof of Theorem 11: One shows, by considering appropriate linear combinations of the equations in the system (A.10), that

$$\lim_{\lambda\to\infty} \Delta v_l^u = \lim_{\lambda\to\infty} \Delta v_h^i = \lim_{\lambda\to\infty} \Delta v_h^u < \lim_{\lambda\to\infty} \Delta v_l^i,$$

which is inconsistent with (22). As noted in Section 4.4, (22) must be satisfied for any pooling equilibrium.

□

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